Transonic flow of moist air around an NACA 0012 airfoil with non-equilibrium condensation*

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Abstract The classical condensation model of water vapor is coupled with the Euler equations to calculate transonic flows of moist air with non-equilibrium condensation. By means of this model, numerical computations are implemented to investigate the aerodynamic characteristics of an NACA 0012 airfoil in transonic flows of moist air at various angles of attack and relative humidities, and the results are compared with those in dry air flows. For different angles of attack considered at 50% relative humidity, the lift decreases 30%—40%. The pressure drag increases when the angle of attack is smaller than 1.4° and decreases when higher than 1.4°. At zero angle of attack, with the relative humidity rising from zero to 90%, the pressure drag increases exponentially. At 90% relative humidity, the pressure drag increases 160%, and self-oscillation takes place periodically and alternately over the upper and lower surfaces of the airfoil. The oscillation is caused by the interactions of local supersonic flow and heat release in the condensation process.

Keywords: NACA 0012, airfoil, moist air, non-equilibrium, condensation.

The transonic flow of atmospheric moist air around airfoils is a fundamental problem of scientific interest. It also has important technological applications in the design of airplanes, helicopters, shock tubes, and wind tunnels. Once the condensation of the water vapor takes place in moist air, latent heat is released and added to the surrounding air. The thermodynamic properties and flow pattern of moist air are thus significantly affected. As a result, changes in the aerodynamic performance of airfoils can be found.

In transonic flows of moist air around airfoils, due to the rapid expansion of water vapor, the condensation presents itself as a non-equilibrium process. The vapor component supersaturates after passing the saturate line, and expands according to the properties of superheated vapor. Consequently, the thermodynamic non-equilibrium process is developed, which can be denoted by supercooling ΔT. When ΔT rises to some determinate value (known as the Wilson point), condensation takes place in the local supersonic region over airfoils. Investigations are focused on the condensation onset on the airfoil surface, the changes of drag and lift coefficients, and the behavior of water droplets after condensation.

The condensation phenomena in transonic flows of moist air might be firstly reported by Schmidt[1]. Many details of humidity effects on aerodynamic characteristics of an NASA supercritical airplane model were investigated by Jordan[2]. Appropriate numerical results for condensing transonic flows over airfoils were published by Robinson et al.[3] Schnerr et al.[4] conducted numerical simulations of transonic flows of moist air based on the Euler equations with a heat source term caused by condensation and related to the condensate mass fraction. A small-disturbance model for steady transonic flow of moist air with non-equilbrium condensation was developed by Rusak et al.[5]

In this paper, a new numerical model is developed for transonic flows of moist air with non-equilibrium condensation. With this model, numerical investigations are implemented to investigate the effects of condensation process on the aerodynamic performance of an NACA 0012 airfoil at various angles of attack and relative humidities.

1 Mathematical model

The condensing flow of moist air can be treated as a two-phase system: the gaseous phase consists of dry air and water vapor, namely the moist air, and the liquid phase is the collection of all the condensate water droplets dispersed in the moist air. For the homogeneous condensation in transonic flows of moist

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air, the diameter of water droplets is smaller than 1 μm. So it is reasonable to neglect the velocity slip between the water droplets and the moist air, and a Eulerian/Eulerian model can be established. The conservation equations of mass, momentum, and energy for the moist air are

$$\frac{\partial \rho_{av}}{\partial t} + \nabla \cdot (\rho_{av} \boldsymbol{U}) = -\rho \dot{m}, \tag{1}$$

$$\frac{\partial(\rho_{av} \boldsymbol{U})}{\partial t} + \nabla \cdot (\rho_{av} \boldsymbol{UU}) = -\nabla p - \rho \dot{m} \boldsymbol{U}, \tag{2}$$

$$\frac{\partial \rho_{av} E_{av}}{\partial t} + \nabla \cdot (E_{av} \boldsymbol{U}) = -\nabla \cdot p\boldsymbol{U} - \rho \dot{m} (h_t - h_{fg}), \tag{3}$$

where ρ_{av} is the density of the moist air, ρ is the density of the mixture of moist air and droplets, $\boldsymbol{U}$ is the velocity vector, p is the static pressure, E_{av} is the total energy of the moist air, h_t is the total enthalpy of the moist air, and h_{fg} is the latent heat. The source terms $-\rho\dot{m}$, $-\rho\dot{m}\boldsymbol{U}$ and $-\rho\dot{m}(h_t - h_{fg})$ are introduced to count the interactions between the gaseous and liquid phases. Here $\dot{m}$ is the condensate mass rate, and it is expressed as

$$\dot{m} = (1 - Y) J \rho_l \frac{4\pi r_c^3}{3} + 4\pi r^2 \frac{dr}{dt} \rho_l N, \tag{4}$$

where J is the nucleation rate, dr/dt is the droplet growth rate. They are determined by the classical condensation theory slightly revised by Kantrowitz and Young[7]

$$J = \frac{q_c}{1+\omega} \sqrt{\frac{2\sigma}{\pi m_m^3}} \frac{\rho_{av}}{\rho_l} \exp\left(-\frac{4\pi r_c^2 \sigma}{3kT}\right), \tag{5}$$

$$\frac{dr}{dt} = \frac{\lambda_g \Delta T}{\rho_l r h_{fg} \left(\frac{1}{1+4K_n} + 3.78(1-\nu)\frac{K_n}{P_{rg}}\right)}, \tag{6}$$

in which ω and ν are two correction coefficients, q_c is the condensation coefficient, σ is the surface tension of water, m_m is the mass of water molecule, ρ_l is the density of water, r_c is the critical nucleation radius, k is the heat transfer coefficient, T is the temperature, and K_n is the Boltzmann constant.

In addition, the status equation of moist air is used to make the Euler equation set enclosure.

$$p = \rho_{av} R T. \tag{7}$$

As noted previously, the velocity slip between the gaseous and liquid phases is neglected, so only additional equations to describe the quantity and size of droplets are needed for the liquid phase. Let N denote the number of water droplets, r the radius, and Y the condensate mass fraction. The conservation law yields

$$\frac{\partial(\rho N)}{\partial t} + \nabla \cdot (\rho N \boldsymbol{U}) = \rho_{av} J, \tag{8}$$

$$\frac{\partial(\rho Y)}{\partial t} + \nabla \cdot (\rho Y \boldsymbol{U}) = \rho \dot{m}. \tag{9}$$

After N and Y are solved the radius r can be determined by the following formula

$$r = \sqrt[3]{3Y/(4\pi \rho_l N)}. \tag{10}$$

In the moist air flow, with the continuous water vapor condensation the humidity decreases, leading to lower partial pressure of water vapor p_v. In order to determine p_v, the distribution of humidity d needs to be resolved. Let $D = d/(1+d)$, we can get $\rho_v = \rho_{av} D$ from the definition of humidity, where ρ_v is the density of water vapor. Based on the conservation law, the governing equation of humidity can be deduced:

$$\frac{\partial \rho_{av} D}{\partial t} + \nabla \cdot (\rho_{av} D \boldsymbol{U}) = -\rho \dot{m}. \tag{11}$$

Now, the transonic flow of moist air with non-equilibrium condensation can be solved with Eqs. (1)—(3) and (7)—(11). The finite volume method and explicit time-marching technique are used. The second-order upwind scheme is adopted in the spatial discretization. For steady flow the temporal first-order scheme is used and for the time dependent problem, the dual time step method is used. The unstructured triangular mesh is employed. The computations are carried out in a circular zone with a diameter of 10 times of the airfoil chord c. The total number of control volumes is about 75000.

In all calculations, only the effects of angle of attack and relative humidity are investigated. The following parameters are kept unchanged: $c = 0.1$ m, $p_\infty = 65600$ Pa, $T_\infty = 259$ K, and $M_\infty = 0.8$.

2 Analysis of the numerical method and results

2.1 Validation of the model

The transonic flow of moist air around the NACA 0012 airfoil at $\alpha = 0°$ and $\phi_\infty = 50\%$ was calculated and compared with the available data in literature to verify the model. Fig. 1 shows the distributions of pressure coefficient C_P on the airfoil surface. Both the present results and those given by Schnerr et al[4]. indicate that the heat release during condensation process leads to higher static pressure on the airfoil surface in front of the shock. Therefore, the shock shifts upstream and its strength are weakened significantly. It can be also noticed that there exists difference between the present results and those proposed by Schnerr et al. for the onset of surface pressure (thereby the onset of condensation) in the flow of moist air. This should be attributed to the difference of condensation models adopted by Schnerr et al. and by the present authors. Up to now some mechanisms of condensation have not been clearly understood, so different condensation models may yield results with some difference.

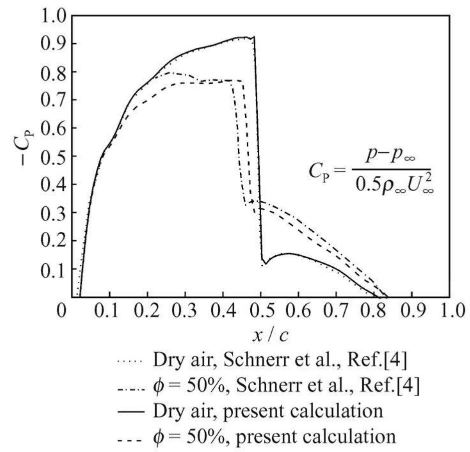

Fig. 1. Distribution of pressure coefficient at zero angle of attack.

2.2 Effects of angle of attack α

The effects of angle of attack were investigated for the transonic flows of moist air around the NACA 0012 airfoil. The relative humidity was kept as $\phi_\infty = 50\%$ and the angle of attack α increased from zero to 4°. The lift coefficient C_L and drag coefficient C_D are shown in Fig. 2. For both flows in dry air and moist air, C_L equals zero at $\alpha = 0°$, as the theory predicts. However, it is not true that C_L always equals zero for flows of moist air at zero angle of attack, and such a case will be discussed in Section 2.4. As shown in Fig. 2, C_L rises almost linearly when α increases. Compared with that in dry air flows, C_L in moist air flows decreases 30%—40%. The drag coefficient C_D rises when α increases. In the range of $\alpha < 1.4°$, C_D in moist air flows is larger than that in dry air flows; when $\alpha > 1.4°$ the situation reverses. At $\alpha = 4°$, C_D in moist air flow decreases about 27% compared with that in dry air flow.

Fig. 3 gives the mach number contours which show the mechanisms leading to the changes of C_L

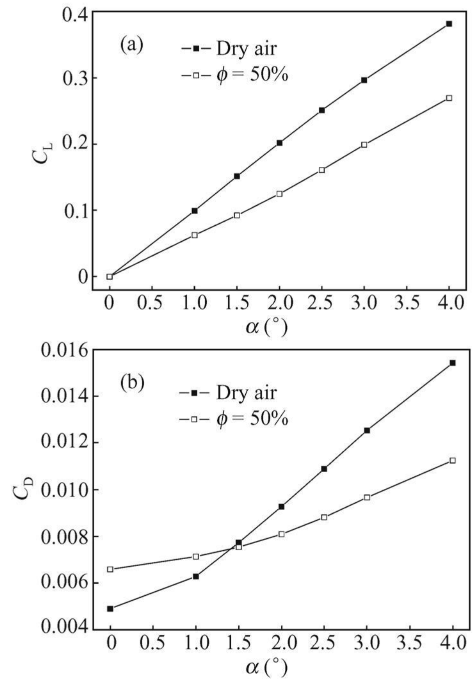

Fig. 2. Lift and drag coefficients. (a) Lift coefficient; (b) drag coefficient.

and C_D. The mach number in front of the shock in moist air flow at $\alpha = 0°$ is smaller than that in dry air flow, and this pattern reverses behind the shock. From the compressible dynamics, it can be known that the surface pressures on two sides of the shock for moist air flow and dry air flow have inverted relationship compared with the mach number. The changes of the static pressure on the surface of the airfoil lead to larger C_D in moist air flow than that in dry air flow at zero angle of attack. At $\alpha > 0°$, the lift is generated. The area-averaged pressure on the upper surface of the airfoil in moist air flow is larger than that in dry air flow. The area-averaged pressures on the lower surface in dry and moist air flows are close due to the slight effects of the condensation near the lower surface. As a result, the lift in moist air flow is smaller than that in dry air flow. The variation of pressure drag stems from a similar mechanism and will be omitted here. In addition, it can be seen that the shock is significantly weakened and shifts upstream in the moist air flows due to the heat release in the condensation process.

2.3 Effects of relative humidity ϕ_∞

The transonic flows of moist air at $\phi_\infty = 30\%$, 50%, 70%, and 90% were calculated and compared with the dry air flow at zero angle of attack. The pressure coefficients on the surface are plotted in Fig. 4.

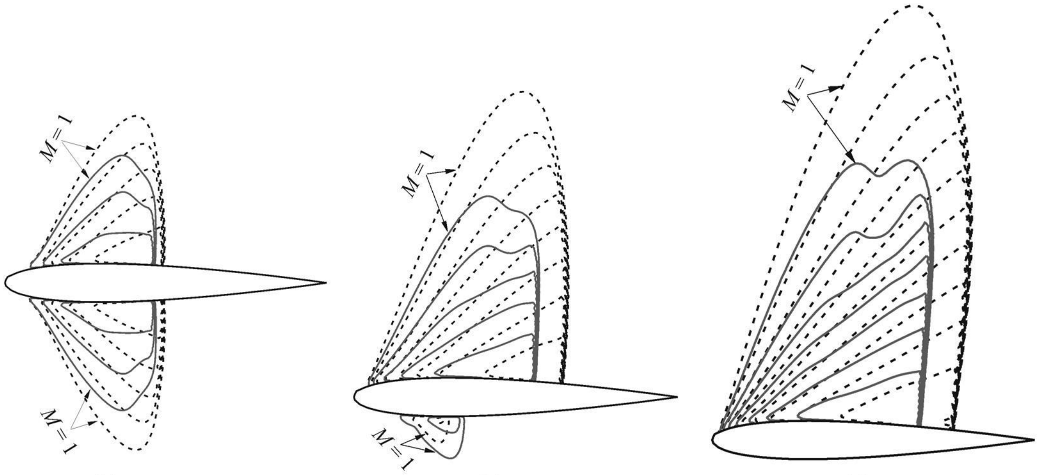

Fig. 3. Contours of mach number ($\Delta M = 0.05$), $\phi_\infty = 50\%$. (a) $\alpha = 0°$; (b) $\alpha = 2°$; (c) $\alpha = 4°$. ——, moist air flow, ---, dry air flow.

It can be seen that with the increasing of ϕ_∞, its effects on C_P increase. Moreover, the self-oscillation in the transonic flow of moist air is observed when ϕ_∞ rises up to 90%. The range of C_P in the oscillating flow is marked in Fig. 4. The variation of ΔC_D (defined as $(C_{D,f>0} - C_{D,f=0})/C_{D,f=0}$) relative to ϕ_∞ in moist air flows is showed in Fig. 5. At $\phi_\infty = 70\%$, ΔC_D is nearly 90%, and at $\phi_\infty = 90\%$, ΔC_D is 160%. It can be seen that ΔC_D increases exponentially when ϕ_∞ rises for this NACA 0012 airfoil. Schnerr et al.[4] conducted investigation for one circular airfoil and concluded that the pressure drag decreases when the relative humidity increases. Therefore, it is clear that the variation of pressure drag in transonic flows of moist air is determined by both the changes of surface pressure due to the heat release in the condensation process and the characteristics of the given airfoil profile. Compared to that in dry air flows, the pressure drag in transonic flows of moist air may increase or decrease for various airfoils.

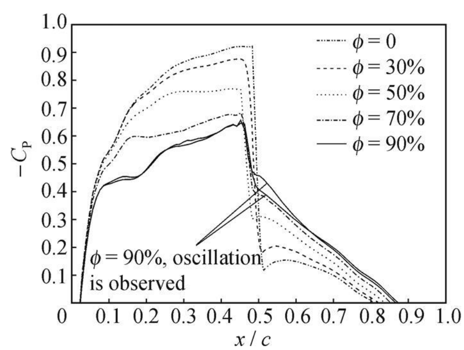

Fig. 4. Pressure coefficient at zero angle of attack.

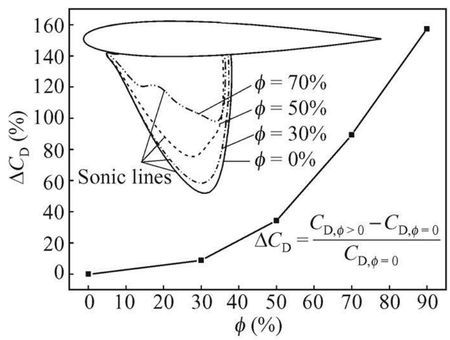

Fig. 5. Increase of C_D at zero angle of attack.

2.4 The self-oscillation in moist air flow

The self-oscillation is observed in the transonic flow of moist air at $\alpha = 0°$ and $\phi_\infty = 90\%$. Fig. 6 shows the time history of C_L and C_D. The frequency f is 1592 Hz. It can be seen that the lift coefficient C_L fluctuates though its time-averaged value is zero. The drag coefficient C_D also fluctuates but the amplitude is small due to the small horizontal component of the oscillating pressure on the thin profile of the airfoil. The time history of pressure coefficients on the position $x/c = 0.5$ for the upper and lower surface are also plotted respectively in Fig. 6. It can be seen that the pressure fluctuates on both the upper and lower surface. The amplitudes are the same but the phase angles are reversed. Fig. 7 shows the time history of mach number contours. It can be seen that the flows near the upper and lower surface oscillate periodically and alternately.

In supersonic flow when the latent heat release is much greater than the critical heat addition (which will reduce the mach number of supersonic flow to 1), the self-oscillation may take place. The mechanism has been studied by Skillings et al.[7] and can be briefly concluded as: the heat release reverts the supersonic flow to a subsonic condition, thus the condensation is weakened and the heat release decreases. As a result, the subsonic flow accelerates to supersonic flow once again, and a new cycle of oscillation starts.

This calculation indicates that the self-oscillation may take place in transonic flow of moist air around airfoils when the relative humidity is high enough.

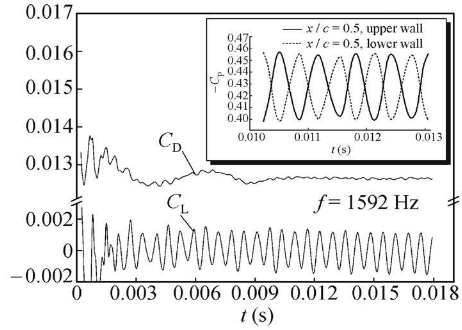

Fig. 6. Characteristics of self-oscillation in moist air flow with condensation. Insert is the pressure coefficients on the walls.

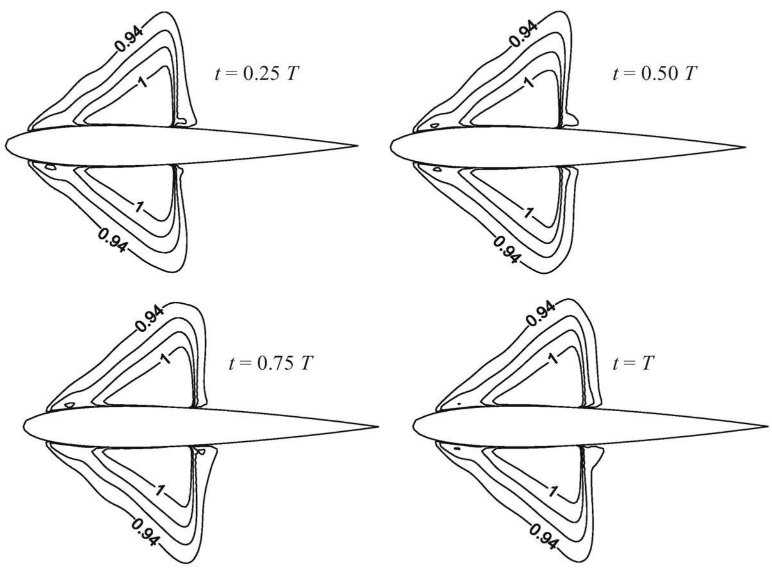

Fig. 7. Time history of mach number contours in moist air flow with condensation. T is the period of the self oscillation.

3 Conclusions

A new model for transonic flows of moist air with non-equilibrium condensation is developed. With this model, the aerodynamic performance of a NACA 0012 airfoil in transonic flows of moist air at various angles of attack and relative humidities is investigated and compared with that in the dry air flows. In the conditions considered the lift decreases significantly and the drag may increase or decrease. The changes of the lift and drag are determined by both the variation of static pressure on the airfoil surface due to the heat release in condensation, and the characteristics of the airfoil. The self-oscillation in transonic flow of moist air is also observed at zero angle of attack and 90% relative humidity. The symmetrical flow pattern regarding to the chord is broken and the fluctuations are found periodically and alternately over the upper and lower surfaces of the airfoil. The flow unsteadiness is due to the interactions between the local supersonic flow and the heat release in the condensation process.

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